TEACHER KNOWLEDGE ACTIVATED IN THE CONTEXT OF DESIGNING PROBLEMS

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The investigation of teachers' knowledge that informs practice in the mathematics classroom is an important area for research. This issue is addressed in our larger research program which is aimed at characterising the complexity and multi-dimensionality of this knowledge. A report on an earlier phase of this program (Butterfield & Chinnappan, 2010) showed that pre-service teachers tended to activate more common content knowledge than content that is required for teaching. We build on this previous work by examining the kinds of knowledge that a cohort of pre-service teachers activated in the context of designing a learning task.

Introduction

Current reforms and debate about improving the quality of mathematical learning are increasingly concerned with the kind of learning experiences teachers can provide for the learners (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010). The quality of these learning experiences in turn depends on teachers' own knowledge and experiences (Ball, Hill & Bass, 2005). There has been a surge of interest in examining teacher knowledge that drives their actions in the classroom. This study is located within this increasing concern with knowledge that is necessary for the support of deep mathematical understanding.

Context for the study

The performance of teachers has come under increased focus as reflected by accreditation requirements of professional bodies. In order to be accredited by professional bodies such as the NSW Institute of Teachers (NSWIT) and the Queensland College of Teachers (QCT) prospective teachers need to demonstrate that they have achieved a set of minimum knowledge and skills. This development has brought a high degree of urgency among tertiary educators to ensure that their programs and teaching modules are aligned with standards identified by such professional bodies. All these clusters of standards have one thing in common, which is that teachers must develop strong content and pedagogical knowledge. This is the focus of the study.

While the Australian National Curriculum is in various states of implementation a common teaching requirement is the consideration of performance against national

standards (ACARA, 2010). This development again has brought the microscope on teaching and teaching knowledge.

Ball, Hill, and Bass (2005) have identified four dimensions of knowledge that are important for teachers to function effectively in a classroom: Common Content Knowledge; Specialised Content Knowledge; Knowledge of Content and Students; and Knowledge of Content and Teaching. These dimensions provide direction for the assessment of teacher knowledge for teaching. The elucidation of this knowledge is somewhat complicated due to the fact that this knowledge is internal. In order to gain insight into this knowledge, it is necessary to externalise the knowledge by providing a range of contexts to elicit this knowledge. It would seem that the richer the context in which the teachers are embedded, the better the quality of teacher knowledge that can be accessed. This logic led us to design a research study in which a cohort of pre-service teachers was asked to develop a complex problem that can be used in Upper Primary classrooms.

Our long term aim is to map the growth of this knowledge during the Graduate Diploma of Education (GDE) program. This study is a follow up of a previous study (Butterfield & Chinnappan, 2010) that was set against the above background concerning teacher knowledge that informs teaching. The results of this study showed that our GDE Pre-Service Teachers (PSTs) tended to access a higher proportion of Common Content Knowledge (CCK) than components of teacher knowledge that are more relevant to their work in the class. Specifically, we found that their knowledge of Specialised Content Knowledge (SCK), Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) were weak. This is not unexpected, as the participants were commencing their studies.

This study is aimed at boosting and assessing the growth of PSTs' knowledge of SCK, KCS and KCT. As described below our lectures and tutorials were modified in order to bring about changes in the above knowledge. This strategy involved guiding the PSTs to construct learning activities that were investigative in nature.

Related literature

Teacher knowledge

Research (Shulman, 1987) on teacher knowledge has spawned a number of studies concerning teacher knowledge and practice (Ma, 1999; Schoenfeld, 2010). In the past decade these studies have attempted to capture the complexity of teacher knowledge under various conditions including that which is played out in the classroom. This body of research has led to a convergence of view that such knowledge is complex and multifaceted. For example, the studies conducted by Ma (1999) showed that teachers need to transform their content knowledge to teach effectively. Concurrent developments in the United States have generated new directions in the way we could conceptualise and study teacher knowledge. Research in the United States has been led by Ball and her associates, which resulted in the development of more refined dimensions of teacher knowledge (Figure 1). The spirit of this research theme has been embraced by others by examining teacher knowledge in a variety of contexts (Mewborn, 2001).

Teaching as problem solving

A major problem for teachers is to design and implement effective learning experiences leading to sound learning outcomes. The problem, defined in this manner, is rather nebulous as there are multiple paths to the solution. If one conceives teaching as a problem solving activity one is open to a range of opportunities for teachers to exhibit and exploit their knowledge. Problem-solving activities involve searching for a solution within a problem space (Newell & Simon, 1972). The nature of problem space and quality of search is a function of the elements in the space. A corollary of this action is that in an open-ended problem such as teaching, the problem space can be expected to be populated by not only more elements but also the search will be supported by the activation of multiple knowledge sources. Thus, it would seem that the kind of knowledge identified by Ball et al. (2005) are better studied in the context of teachers designing problem-solving activities that can be subsequently used to engage learners. In the present study we adopt this approach.

Conceptual framework

Data analysis and interpretations were guided by the following schematic-representation of teacher knowledge for teaching mathematics (MKT) (Figure 1) (Hill, Ball & Schilling, 2008, p. 174).
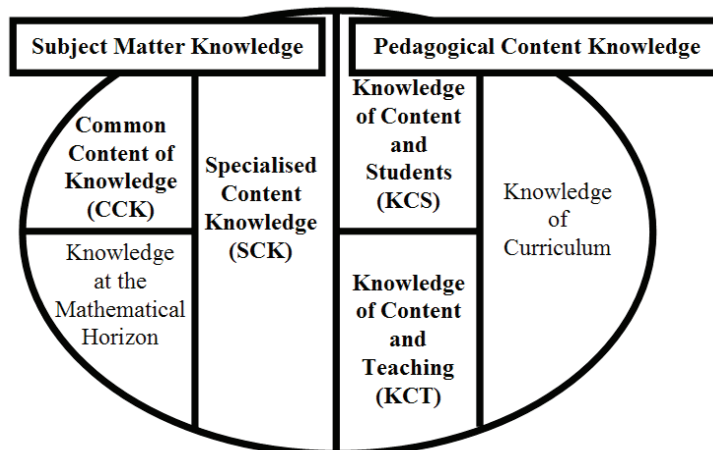

Figure 1. Schematic representation of teacher knowledge for teaching mathematics (MKT).

Four dimensions are defined:
- *Common Content Knowledge (CCK)*: Mathematical knowledge and skill possessed by a well educated adult.
- *Specialised Content Knowledge (SCK)*: Knowledge of how to: use alternatives to solve a problem; articulate mathematical explanations; demonstrate representations.
- *Knowledge of Content and Students (KCS)*: Knowledge that combines knowing about mathematics and knowing about students. Knowledge of how to: anticipate what students are likely to think; relate mathematical ideas to developmentally appropriate language used by children.

- *Knowledge of Content and Teaching (KCT)*: Knowledge that combines knowing about mathematics and knowing about teaching. Knowledge of how to: sequence content for instruction; determine instructional advantages of different representations; pause for clarification and when to ask questions; analyse errors; observe and listen to a child's responses; prompt, pose questions and probe with questions; select appropriate tasks.

Focus questions

The aim of the study was to examine the quality of SCK, KCS and KCT that was activated by a cohort of Pre-service Teachers (PSTs) in the course of designing a problem.

The above aim is reflected in the research questions, seeking

1. evidence of PSTs activating SCK in the context of designing a problem;
2. evidence of PSTs activating KCS in the context of designing a problem;
3. evidence of PSTs activating KCT in the context of designing a problem; and
4. a correlation between the quality of the problem representation and activation of SCK, KCS and KCT.

Methodology

Participants

A cohort of 26 Graduate Diploma of Education students in the final semester of their one-year degree participated in the study. The cohort had completed a numeracy course prior to this mathematics subject, and had also completed professional experience in schools.

Task

Pre-service teachers were required to work in pairs to design a mathematical problem suitable for Upper Primary school children. In designing the task the PSTs were instructed to develop a problem that is isomorphic to the Truss Bridge Problem (Butterfield & Chinnappan, 2010).

Procedures

PSTs were provided with a range of prompts and supports in both the lectures and tutorials before they designed their own problem. The Truss Bridge Problem (TBP) (Butterfield & Chinnappan, 2010) was utilised in a number of tutorials and lectures. This involved discussions about the different problem representations of TBP and how such representations could permit or hinder transfer to other problems by learner. The TBP also highlighted the role and the development of a child's knowledge and skills in Number, Patterns and Algebra, and Space. In addition, we examined the use of appropriate materials and methods (including technology) to solve problems of this type and likely difficulties children could encounter. The TBP, therefore, provided PSTs with a stimulus for hands on activities and reflection on the knowledge components required in subject matter and pedagogy. The PSTs were also given multiple opportunities to explore and solve the TBP. Thus in designing their own new problems we are comfortable in assuming that the PSTs are cognisant of the multiple solution paths and associated representations of the problems.

Representations of TBP and Coding

The TBP (Figure 2) that was developed in the previous study (Butterfield & Chinnappan, 2010) has a certain structure reflecting a hierarchy in the way that it can be represented. The hierarchy is as follows:

1. Concrete – uses concrete materials or physical means to provide a solution
2. Sequential – uses a table to provide a sequential, linear set of solutions
3. Generalisation – describes the pattern that can be used to provide a solution to any given number
4. Transferability – describes how the pattern can be used to solve similar problems

Figure 2. Truss bridge problem.

The hierarchical structure in the Truss Bridge Problem guided us in developing instructions for problems with similar structures. This structure also provided a coding scheme to rate the quality of task developed by the students.

Sources of data

There are two sources of data for the study. The first source involved examining the quality of the problem designed by the students. The coding system is based on the hierarchy of the TBP.

The second source of data involved determining instances of activation of three categories of knowledge (SCK, KCS, KCT). In order to generate this data we analysed PSTs' reflective reports, digital presentations and their responses to questions about the likely difficulties and useful ways to develop children's understanding. The researchers independently coded these instances in order to establish inter-coder agreement.

Results

Participants provided a range of problems that could foster algebraic thinking. The problems designed by student pairs are outlined in Table 1. All problems lend themselves to an analysis of problem representations along the dimensions of TBP.

Table 1. Description of problems.

Problem	Description
Tricky Trapezium Tables	Number of children seated at a row of trapezium-shaped tables
Multistorey Car Park	Number of beams to construct the front of a multistorey car park
Stair	Number of rail posts for a flight of stairs
Stadium	Number of seats in a stadium
Pig Pen	Number of fence panels in a row of pig pens with shared walls
Jack – In – The - Box	Number of exposed body parts with each wind
Dragon	Number of triangular scales per each body part
Terrace Houses	Number of windows in a row of terrace houses
Fence Posts	Number of fence posts in a rectangular paddock
Path Pavers	Number of pavers in patterned path
Angle Sums	The sum of angles in regular shapes
Hay Stack	Number of cylindrical bales in hay stacks
Mosaic Frame	Number of tiles in a frame with coloured corners

Hierarchy of representations for selected problems

Problem Sample 1

An example of a problem coded 2 for problem representation is the Pig Pen problem (see Figure 3). In this problem PSTs did not identify the potential to generalise the pattern to any number of fence panels.

How many fence panels are needed to construct these pig pens*?*

Figure 3. Pig pen problem.

The PSTs stated that the children should complete the provided table (see Table 2) and that as teachers they would like their students to communicate, "I saw that the numbers on the bottom line are going up by three". Here the PSTs were able to identify only the sequential patterns.

Table 2. Pig pen problem worksheet sample.

No of pens	1	2	3	4	5
No of panels	4	7	10		

Problem sample 2

An example of a problem coded 4 for problem representation is the Tricky Trapezium Tables (see Figure 4). The problem enables students to generalise and transfer that pattern to a new problem context. The PSTs stated that "generalisations enable students to recognise that similar problems have a common algebraic basis". To support this statement the PSTs wrote that:

when a child sees the Truss Bridge Problem (see Figure 2) they would say this could be solved by two times the number of triangles plus one, which is the same way to solve the number of people sitting at different shaped tables. For example, the number of people seated around trapezium-shaped table could be determined by counting the number of trapeziums multiplied by three plus two (Number of people = $3n +2$). This reasoning can be applied to squares.

This type of thinking that resulted in generalisation has been argued to lie at the foundation of algebraic thinking (Bobis, Mulligan, Lowrie, & Taplin, 2004).
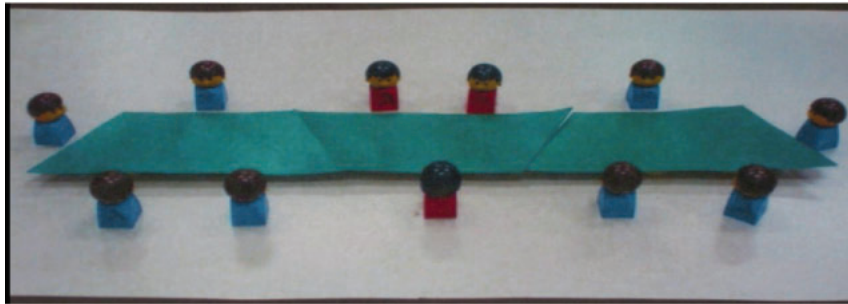

Figure 4. Tricky trapezium tables problem.

In order to generate data that are relevant to research questions 1-3, we analysed the frequency of instances. The mean and standard deviations of this analysis for the four problem representations are given in Table 3.

Table 3. Problem representation and teacher knowledge.

Problem Representation		SCK	KCS	KCT
1	Mean	12.80	7.80	.60
	Std. Deviation	6.22	4.02	.54
2	Mean	14.50	10.50	1.50
	Std. Deviation	2.12	.70	2.12
3	Mean	18.00	17.50	2.50
	Std. Deviation	11.31	9.19	2.12
4	Mean	31.75	25.50	5.00
	Std. Deviation	2.75	4.04	1.41
Total	Mean	19.69	15.15	2.39
	Std. Deviation	9.95	8.90	2.25

We note the accessing of a higher proportion of SCK followed by KCS and KCT. This pattern is also evident within each representation. There is a significant difference between the number of instances of KCT and the other two categories of knowledge across all four categories of problem representations.

Table 4 shows results of correlation analysis among the four variables. While all three knowledge components are highly positively correlated with Problem

Representations, we note KCS and KCT have higher indices. Thus, there was support for our contention that a qualitatively superior problem representation will involve a higher degree of activation of SCK, KCS and KCT (Research question 4).

Table 4. Correlation analysis.

	SCK	*KCS*	*KCT*
Problem Representation	0.81**	0.88**	0.84**

*** Correlation is significant at the 0.01 level (2-tailed).*

Discussion and implications

The previous study showed that student teachers both individually and as a group tended to activate more CCK component of their subject-matter knowledge of mathematics than SCK. The results were consistent with our expectation that as beginning teachers their content knowledge of mathematics, robust though this might be, would not be translated into forms that were more akin to teaching mathematics to children.

The thrust of this study was to map developments in PSTs' teacher knowledge as a consequence of exposing them to a teaching approach that focused on the design of problems. These teachers had also completed two sessions of their professional experience in the school setting. Thus, our expectation was that the classroom experiences and our guidance in designing problems for deep mathematical learning would assist them to reveal a higher incidence of activation of not only SCK but also understanding of student learning and the demands of teaching via an enhanced body of KCS and KCT.

The results do support our contention that having PSTs design rich learning activities would increase their knowledge and activation of SCK, KCS and KCT. Designing problems that will be used to support children's learning requires a level of sophistication in teachers' conceptualisation of the problem environment as shown by the range of problems in Table 1. The corollary here is that teachers have to understand the mathematics that underpins that activity and insights into how children will grasp the problem. We contend that the complexity of the problems teachers have been asked to design have provided multiple points at which teachers could connect with and activate knowledge relevant to the three categories of knowledge.

While all three knowledge categories were positively correlated with the quality of problem representation, the highest correlation was evidenced with KCS which involved teachers understanding learners. It would seem that problem posing activities could be used to enhance the development of KCS, a point that was alluded to by Chinnappan and Lawson (2005).

Results indicated that (Table 3), a significant number of the participants tended to design problems that from a representational viewpoint were somewhat weak. This group either constructed the physical model of the problem or merely provided a table with numbers indicative of growing dimensions. For example, in Figure 4, student teachers could indicate the growth in number of panels per pen for a small number of pens (1-5). That is, the only pattern they could identify is numbers increasing in threes without being able to extract the *general* pattern that shows the relations between pens and panels. This limitation in the quality of representation, we argue, is the consequence

of over reliance on the accessing of procedural knowledge. This outcome is consistent with that reported by Capraro, Capraro, Parker, Kulm, and Raulerson (2005).

A limitation of the present study is that we did not give prominence to KCT as we assume that this is more accessible in real-life teaching contexts. Future studies should focus on this issue. Also, we acknowledge that it is difficult to generate a complete picture of pre-service teachers' pedagogical content knowledge within the confines of one assessment task that was completed for a university subject. Further studies with a greater variety of such tasks might provide more opportunities to examine this knowledge.

References

Australian Curriculum, Assessment and Reporting Authority [ACARA] (2010). *Draft Australian Curriculum*. Sydney: Australian Curriculum Assessment and Reporting Authority. Retrieved 26 January 2011 from http://www.australiancurriculum.edu.au/Home.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29*(3), 14–22, 43–46.

Butterfield, B., & Chinnappan, M. (2010). Walking the talk: Translation of mathematical content knowledge to practice. In L. Sparrow, B. Kissane, & C. Hurst (Eds.), *Shaping the future of mathematics education* (Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia, pp. 109–116). Fremantle, WA: MERGA.

Bobis, J., Mulligan, J., Lowrie, T., & Taplin, M. (2004). *Mathematics for children: Challenging children to think mathematically*. Sydney: Prentice Hall.

Capraro, R., Capraro, M, Parker, D., Kulm, G. & Raulerson, T. (2005). The Mathematics Content Role in Developing Preservice Teachers' Pedagogical Content Knowledge, *Journal of Research in Childhood Education, 20*(2), 102–118.

Chinnappan, M., & Lawson, M. (2005). A framework for analysis of teachers' geometric content knowledge and geometric knowledge for teaching. *Journal of Mathematics Teacher Education*, 8(3), 197–221.

Hill, H. C., Ball, D., & Schilling, S. (2008). Unpacking "pedagogical content knowledge"; Conceptualising and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372–400.

Ma, L. (1999), *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: LEA.

Mewborn, D. (2001). Teachers content knowledge, teacher education, and their effects on the preparation of elementary teachers. *United States Mathematics Education Research Journal, 3*, 28–36.

Newell, A., & Simon, H. (1972). *Human problem solving*. New Jersey: Prentice-Hall.

Schoenfeld, A. H. (2010). How We Think: A Theory of Goal-Oriented Decision Making and its Educational Applications. New York: Routledge

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *American Educational Research Journal, 15*(2), 4–14.